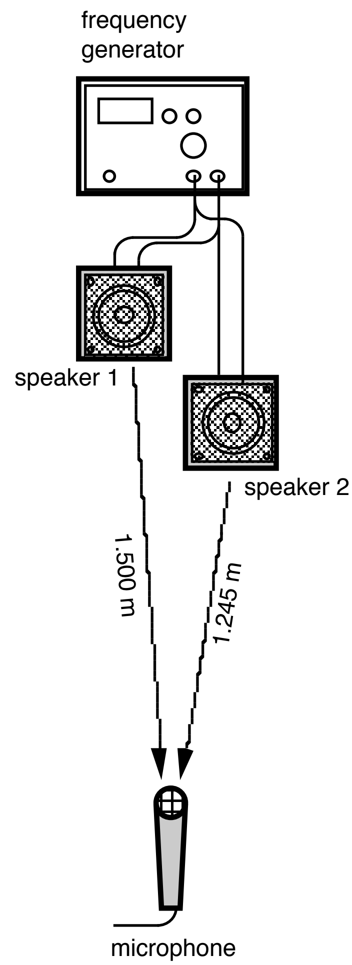
Examples

1.A frequency generator in DL creates a sinusoidal signal:

$$Signal(t) = A\sin\left(\frac{2\pi t}{(0.00025 \text{ s})} + \frac{\pi}{2}\right).$$



This generator is connected to two speakers. However, speaker 2 has its wires switched, with respect to the correctly wired speaker 1. Assume that the sound waves generated by these speakers are 1D.

(a) What are the harmonic sound wave functions for the sounds produced by speaker 1 and for speaker 2?

(b) At time $t = 0$, what are the total phases Φ_1 and Φ_2, and what is the difference in total phase $\Delta\Phi$, at the location of the microphone? Is there constructive or destructive interference? Use a phase chart to organize your answer.

Solution

Since these are sound waves, the wavelength l of the sound waves created by speakers 1 and 2 is:

$$\lambda = \frac{v_{wave}}{f} = v_{wave}\text{T} = \left(340 \ \frac{\text{m}}{\text{s}}\right)(0.00025 \text{ s}) = 0.085 \text{ m}.$$

The harmonic wave functions for the sounds coming from speakers 1 and 2 are then given by:

$$\Delta P_1(x,t) = A\sin\left(\frac{2\pi t}{(0.00025 \text{ s})} - \frac{2\pi x}{(0.085 \text{ m})} + \frac{\pi}{2}\right);$$

$$\Delta P_2(x,t) = A\sin\left(\frac{2\pi t}{(0.00025 \text{ s})} - \frac{2\pi x}{(0.085 \text{ m})} + \frac{3\pi}{2}\right).$$

Note the (-) before each x term, as both sound waves propagate outwards from their respective speaker (using x to represent any direction outwards from the speaker). Also note that x is the distance measured from the source to a specific position, which will differ for the two speakers.

The microphone is located 1.500 m away from speaker 1, and 1.245 m away from speaker 2.

Phase chart for interfering waves

	$\frac{t}{T}2\pi$	$\pm\frac{x}{\lambda}\,2\pi$	ϕ	Φ	Unit Circle/ Interference
Speaker 1	0	$-\frac{1.5m}{0.085m}2\pi$	π/2	-17.4*2π	
Speaker 2	0	$-\frac{1.245m}{0.085m}2\pi$	3π/2	-13.9*2π	
Difference	0	-3*2π	-π	-3.5*2π	D

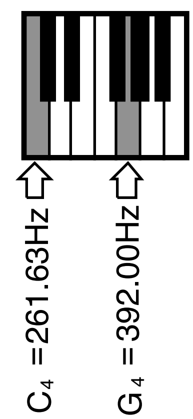
For t= 0s x_1= 1.245m , x_2= 1.500m

The interference is destructive at the location of the microphone since the difference in total phase is a half number of 2π apart. One could also write ΔΦ=-7π and say the total phase difference is an odd number of π apart.

2. The frequencies for the notes C_4 and G_4 are 261.63 Hz and 392.00 Hz, respectively. These notes are played simultaneously to produce a simple chord (try this out if you can get to a piano or keyboard). Calculate the following:

(a) The carrier frequency (which would be perceived as pitch) of these superposed notes.

(b) The subjective beat frequency of these superposed notes.



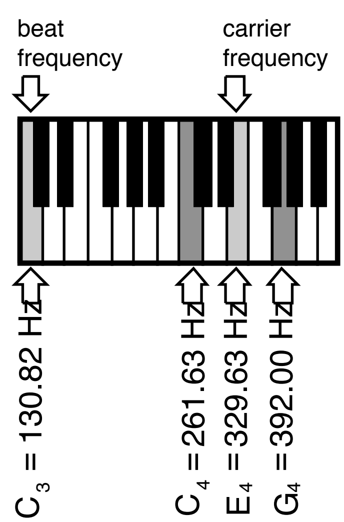
Solution

The carrier frequency is the average of two frequencies in this chord:

$$f_c = \frac{(f_1 + f_2)}{2} = \frac{(261.63 \text{ Hz} + 392.00 \text{ Hz})}{2} = 326.82 \text{ Hz}.$$

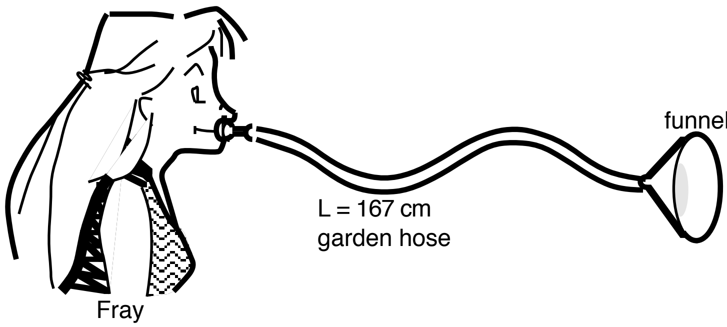
The beat frequency is the difference between the two frequencies in this chord.

$$f_b = |f_1 - f_2| = |261.63 \text{ Hz} - 392.00 \text{ Hz}| = 130.37 \text{ Hz}.$$



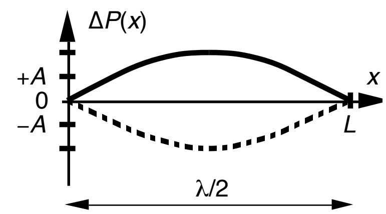
If you look up these frequencies, you will find that playing the notes C_4 and G_4 together in a chord will result in a superposed sound with a pitch of approximately E_4 (329.63 Hz), but with an amplitude modulation of approximately C_3 (130.81 Hz). Try out some different chords, both audibly and mathematically (*cf.* the piano key frequencies in the following Appendix section).

3. Fray puts a trumpet mouthpiece on a 167 cm long garden hose that has a funnel on the other end. If this homemade trumpet behaves like an open-open pipe (and thus like a string fixed at both ends), calculate the three lowest frequencies that she can play on it.



Solution

The fundamental frequency is when a half-wavelength standing wave is produced in this open-open pipe of length L = 1.67 m (note that it doesn't matter that this pipe is straight or crooked):



$$L = \frac{\lambda_1}{2},$$
$$\lambda_1 = 2L = 2(1.67 \text{ m}) = 3.33 \text{ m},$$

such that the fundamental frequency is then:

$$\lambda_1 f_1 = v_{wave},$$
$$f_1 = \frac{v_{wave}}{\lambda_1} = \frac{340 \ \frac{\text{m}}{\text{s}}}{3.33 \text{ m}} = 102.1 \text{ Hz},$$

where v_{wave} is 340 m/s, the speed of sound waves in air. (If we were concerned with standing waves on ropes, we would instead use the speed of rope waves.) The next two higher frequencies than can be played on this open-open pipe are then:

$$f_2 = 2f_1 = 2(102.1 \text{ Hz}) = 204.2 \text{ Hz},$$
$$f_3 = 3f_1 = 3(102.1 \text{ Hz}) = 303.3 \text{ Hz}.$$

(These correspond to approximately G_2# (103.83 Hz), G_3# (207.65 Hz), and D_4 (293.66 Hz). Due to the variation on the speed of sound waves due to altitude, temperature, humidity, *etc.*, most real instruments are "tuned" to produce the correct frequency tones by slight adjustments to its length.)